Glimpses of Vedic Geometry

(The following is the "Introduction" to the unpublished book on Vedic Geometry by Prof. K.V.Krishna Murthy)

रमा ब्रह्माणी संकलित ललित द्रव्य धनिको , शिवस्यापांगेन व्यवकलित दोषर्ण विततिः ।
गुणी श्लाघापात्रं विहित बुध धी भागहरणो , सुधीवर्गे नूनं मथितगणितो यस्स जयति ।।

He becomes richer by ***adding*** the finer qualities of both Lakshmi and Saraswathi to himself. His bad qualities have been ***subtracted*** from him by the kind look of Shiva. Since his good qualities have increased, admirations from others have ***multiplied*** for him. He ***divides*** the hearts of scholars and keeps them with him. i.e., all these scholars appreciate him. These are the achievements of good-hearted person, who churned the ocean of mathematics and he excels all scholars.

This is a verse by the author in praise of Mathematics and Mathematicians.

Since Mathematics is none other than the logic, wherever and whenever Mathematical sciences advance, the growth of Physical sciences follows.

This is a fact established by history. In order to assess the state of scientific development of any country at a given time, generally it may be adequate to study the state of mathematics of the time. We can infer the state of other sciences from that.

Just as Mathematics, is for Physical sciences, so is logic for philosophical knowledge. Wherever logic develops, math too flourishes there. In fact, there is only one difference between math and logic. Logic is expressed in the local language, whereas math is expressed in numbers and lines.

Ancient Indians, who had a spectrum of sciences for their credit, valued mathematics with the same reverence. Here is an evidence for this, form a work called Vedanga Jhyotisha, which belongs to a period older than 1200 B.C.

यथाशिखा मयूराणां , नागनां मणयो थमा ।
तथा वेदांग शास्त्राणां , ज्योतिषं मूर्धनि स्थितम् ।।
(गणितं मूर्धनि स्थितम्) (वेदांग ज्योतिषे)

Just as the natural feather on the head of a peacock and just as gems on the head of a divine serpent, Jyotisha - the science of astronomy is placed on the head of the other sastras - so says the well known Vedanga Jyotisha.

Another version of this sloka reads **गणितं मूर्धनि स्थितम्** infact these versions do not make much difference, for the terms *Jyotisha* and *Ganita* are almost synonyms.

In Bhagavatgita the lord says -

कालः कलयतामहम् ।

कलयतां कलनं गणनं कुर्वता

is the meaning given by Sri Sankaracharya. **कलनं, संकलनं, व्यवकलनं** are the basic forms of **गणनं . संकलनं** Means addition, **व्यवकलनं** means subtraction. Simplified addition is multiplication and simplified subtraction is division. Whatever may be the extent of development, mathematics can never go beyond these four operations, to be more precise, beyond these two operations.

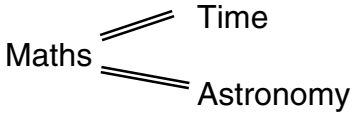
Hence, we can easily infer that Kaala and Ganita are not different from each other. Jyotisha is the science of Kaala. Hence, Jyotisha and Ganita can not be different. Hence we can write:

Time = Astronomy = Mathematics.

But you can ask one question. Time is represented by numbers, where as jyotisha deals with geometrical figures. Then, how can these two be equal? When these two are not equal, how can we say

Time = Astronomy = Mathematics

Your question may be correct only to some extent. Astronomy is the science of time and so, some may not like to say that time = Astronomy even then, they cannot deny the equations.



Why not?

This is because, mathematics is a science in which there are two important and unseperable branches, viz.,

1. Mathematics of numbers &
2. Mathematics of space.

The maths of time deals with numbers and the maths of space deals with lines, whether straight or curved.

Since time and space are inseparable, you can not separate these two branches of mathematics. Since the most ancient and the most modern concepts accept that, in the ultimate, time and space are not different, these two branches of mathematics also cant be different, of course , in the ultimate.

The "ultimate" is thoroughly discussed in the Vedas from different angles and hence, the Veda had to deal with the numbers, the space and the mathematics which embodies both of them.

Stray numbers can be used and utilized by even primitive societies, but numbers as a system can be utilized only by a developed society. In fact, history proves to us that, in any period of time, where the numbers are used in a more systematic way, the better is the civilization of that society as a whole.

Even though we are not exactly sure of what a Vedic period is, whatever it may be, we can estimate its civilization basing on the number systems available in the Vedas.

Pythagoras, the celebrated philosopher and mathematician of 3rd century B.C, tried to evolve a numbering system, to count the particles of a sand in a given jar of sand and wrote his thesis - "*The calculus of sand*". But, unfortunately, he could not develop a perfect decimal system of numbers, because he could not think of "*Zero*" at that time.

But thousands of years before that, we find a full-fledged decimal system of numbers in the Veda mantras:

एका च 10^0	**दश 10^1**	**शतं च 10^2**	**सहस्त्रं 10^3**	**चायुतं च10^4**
नियुतं च 10^5	**प्रयुतं 10^6**	**चार्बुदं च 10^7**	**न्यर्बुदं च 10^8**	
समुद्रश्च 10^9	**मध्यं 10^{10}**	**चांतश्च 10^{11}**	**परार्धश्च 10^{12}**	

Which gives the values of 10^0 to 10^{12}. Please note that the value of 10^0 is given as "1" in the sequence.

This is not a rare or strange reference from Veda.

सकृत्ते अग्ने नमः
द्विस्ते नमः
त्रिस्ते नमः
चतुस्ते नमः
पंच कृत्वस्ते नमः
दश कृत्वस्ते नमः
शत कृत्वस्ते नमः
आसहस्त्र कृत्वस्ते नमः
अपरिमित कृत्वस्ते नमः -कृ .य.वे..

" Oh Lord Agni! Prostrations to you once, twice, thrice, four times five times, ten times, hundred times upto thousand times and unlimited number of times". Here we find a definite pattern of progression of numbers.

In the well known *Chamakadhyaya* of *Krishna Yajurveda*, the mantra

एकाचमे तिस्त्रश्चमे, पंचचमे . . . .
gives two sequences of numbers .

VIZ., 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31
And 4, 8,12, 16, 20, 24, 28, 32, 36, 40, 44, 48.

The first one is a sequence of odd numbers from 1 to 31 for the second one is not a sequence of simple even numbers. We can observe that these sequences follow the formula. $x_n + x_{n+1} = y_1$; x being the member of the 1st sequence and Y , of the 2nd sequence.

Eg: 1 + 3 = 4; 3+5=8 upto the 12^{th} place.

Thus we find an intricate and perfect systematic use of numbers in the Vedas.

Added to this, perfect discretions of cosmological events are found in the Vedas. The famous *Nakshatreti Prakarana* of *Yajurveda*, the cosmological details given in *Rigveda*, are more than enough to prove this.

As we entered the post Vedic Ancient Literature, there are four important places, where we find extensive use of mathematical formulas.

1. The Sulba Sutras
2. Vedanga Jyotisha
3. Chandas Sastra
4. Tantra.

शिक्षा व्याकरणं छंदो
निरुक्तं ज्यौतिषं तथा ।
कल्पश्चेतिषडंगानि
वेदस्याहुर्मनीषिणः ।।
शिक्ष, व्याकरण, छैदस्, नारुक्त , ज्यौतिष,कल्प

these are the six Auxiliaries to Veda - so say the wise ones.

1. Of these, the above referred Sulba Sutras are an integral part of Kalpa.
2. Vedanga Jyotisha is the essence of jyotisha, the science of time and the science of cosmological bodies.
3. Chandas is the science of meters of letters.

4. Tantra is a science and art which tries to simplify Vedic rituals, for the benefit of the less privileged beings.

Of these four –

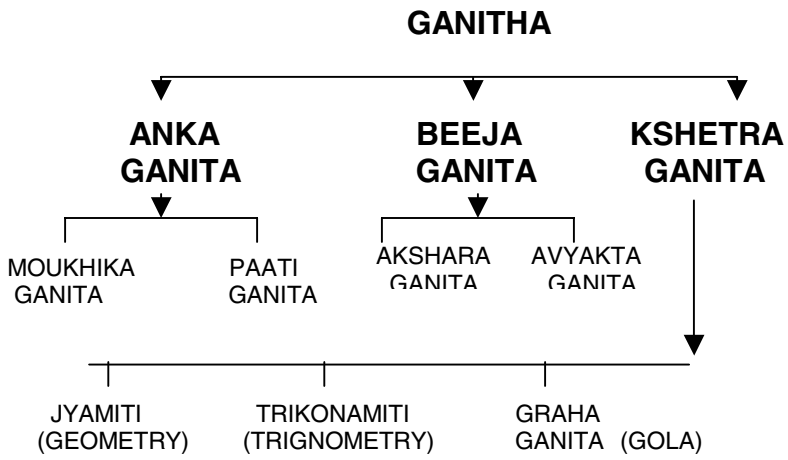
1. Sulba Sutras use maths in the context of the construction of different types of **यज्ञशाल** and **यज्ञवेदिक**

2. Jyotisha uses maths, at every step.

3. Chandas uses maths, when it has to deal with the permutations and combinations of the meters and

4. Tantra uses maths when it translates the divine energies into geometrical figures called Yantras.

So, to understand Vedic Mathematics, we have to study all the four subjects, referred above. But studying them directly is not a simple thing because they are not direct textbooks of mathematics. But a lot of mathematics is used there. So, to make mathematics simpler and easier to understand, the later scholars started dividing mathematics into several sub-branches, thus—



PROMINENT VEDIC MATHEMATICIANS ;

Naarada, Kapila, Bodhayaana, Aapastamba, Lagadhaa etc., are said to be the prominent mathematicians of the ancient periods.
While

Aaryabhatta of	5th Century A.D
Varaahamihir of	6th Century A.D
Brahmagupta of	7th Century A.D
Sreedhara of	8th Century A.D
Mahaaveera of	9th Century A.D
Bhaaskara of	12th Century A.D

are some of the very prominent mathematicians of medieval periods.

These masters have developed wonderful methods of teaching mathematics to the youngsters. But, in the process of doing so, unfortunately, it seems that they had deviated from the age-old Vedic techniques, which were prevalent during the Ancient periods.

In the recent past *Jagadguru Bhaarati Krishna Teertha* of Dwaraka Sankaraachaarya Peetha, revived 16 fundamental Vedic statements of mathematics and proved how these 16 formulas could be used for almost all branches of mathematics. His writings were published under the title " Vedic Mathematics" in 1965 for the first time and eye brows of international mathematicians rose up at once, since the methods annunciated in the Vedic mathematics are unbelievable, simple and perfect, as compared to today's so called advanced western mathematics.

Hence *Bhaarati Krishna Teertha's* Vedic Mathematic is acquiring more and more importance, internationally. So, we also shall start our study of mathematics, through this saint.

Before actually entering into the area of Vedic Mathematics, let us understand, at least, a little bit of the **" THEORY BEHIND VEDIC MATHEMATICS** ".

According to the Vedic concept, there is no fundamental difference between a number and a line. Every number is represented by a line in the space and every line in the space is represented by a system of numbers, (called equations).

For example:

The number one (1) is represented by a straight line segment of unit one length.

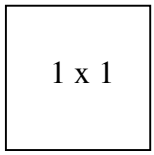
1+1 is represented by a similar line of two units.

1-1 is represented by a line of no units, i.e., a single point (Bindu)

Thus 1,1+1, 1-1 - represent segments of a single dimension in a plane. (of course, a *'Bindu'* can be a part of any plane or dimension)

Now, consider 1 x 1 = 1. But, this one cannot be represented in a straight line. Only a square of one unit length in all the sides can represent this number "One"

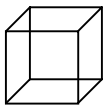
So 1 x 1 = 1



Similarly $\frac{1}{1}$ is also equal to 1. But the above square cannot represent it. It is represented by a straight line only

$$\overline{oneunit}$$

But, 1 x 1 x 1=1 is represented by cube of unit 1



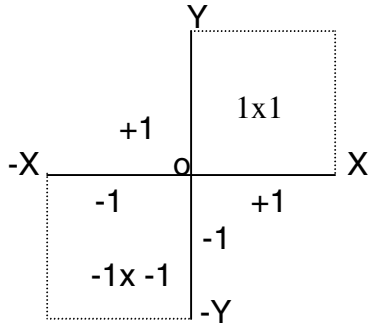
That means; as you go on multiplying numbers, your dimensions in the space also go on multiplying.

Since we agree to accept only 3 dimensions in the space, we agree to ignore more number of dimensions, as far as mathematics is considered.

Now consider –1x(–1) = +1

Can this be represented by a square of one unit, which is just mentioned? The answer is not so simple

why? We shall see.



1 x 1 and –1 x –1 may be equal number wise,

but space wise, they are only similar, but not identical.

For the same reason, square root of $\sqrt{1}$ cannot be a single value. It should give rise to two values, one corresponding to the x y-axis (1^{st} quadrant) and the other corresponding to the x, -y axis.(4^{th} quadrant)

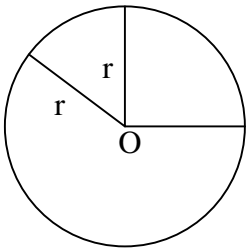
similarly $\sqrt[3]{1}$ (cube root of one) yields three values corresponding to the x,y,z axes .

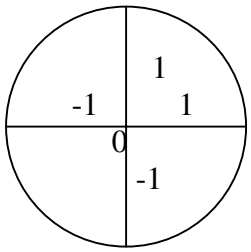
They are 1x1x1, -1x(-1)x1, 1x(-1)x(-1)

Now, think of a circle. Circle is a line, which satisfies the equation

ox = r where ‘o’ is the center of the circle and ‘r’ is a constant.

Suppose r = 1, then, can there be a different circle for r=-1 value? No. Hence, we agree to say that 'r ' can never be –ve. That is why, our Vedic mathematicians said, a line is more perfect than a number. The sutra given by sri Kalyaanaananda Bhaarati, a great exponent of taantrik mathematics is ---





पद - बिजा - संख्या - रेखाणामुत्तरोत्तरं बलीयः

This means, as compared to the word "one", the representative letter "x" is better; compared to this, the symbol"1" is better; and compared to this, the line on the "ox" axis is better.

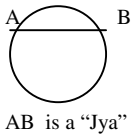
Thus, our Vedic seers see mathematics simultaneously from both the angles of number and space and because of this, they could evolve very simple mathematical procedures.

As explained earlier one of the branches of mathematics, Kshetra Ganita, is further classified as Jyamiti, Trikonamiti, Graha or (gola) ganita.

KSHETRA GANITA:

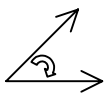
JYAAMITI (geometry)	TRIKONAMITI (trigonometry)	GRAHA (gola) Astronomy (Three Dimensional Geometry

A. "JYAA" means a card , like this



AB is a "Jya"

B. Kona means an angle like this



C. Generally Jyaamiti and Trikonamiti are mixed in our ancient text books

D. We shall study these two, briefly through Sulba sutras.

* What are Sulba sutras?

* Sulba sutras are not textbooks of mathematics or geometry. They are the textbooks which describe different types of “YAGNAS,” “YAGAS” and procedures related to them. In that context, they had to give clues to construct altars “YAGNA VEDIS” and “YAGNA SAALAAS” of different shapes. For that, they had to refer to some of the mathematical formulas which were popular in those days.

* The instructions were being given to a common, uneducated, coolly like person who is at the work of constructing the “VEDIS”. Hence, sulba sutras took care, to see that no unnecessary complications creep into their explanations or procedures.

* The word “SULBA” means a thread or a rope. The whole procedure of constructing rectangular, square, triangular, circular, semicircular and such other shapes of the “YAGNA KUNDAS,” the sages like Apastamba, Bodhayana, Satyaashaada etc., took the aid of only two things viz., sulba (a thread) and scale. That is all. No meters to verify right angles, no compass to draw circles, no angulometer to measure angles, nothing of the sort. But, still, they could describe such complicated procedures like constructing a square or rectangle of a given area and then constructing a circle or semi circle of the same area etc. All these things they could accomplish with the help of a thread and a scale only.

* In addition to this, they took opportunity to describe the values of π, $\sqrt{2}$, $\sqrt{3}$ etc.

* Now, we shall have a few glimpses of those astonishing ancient “SULBA SUTRAS”.