AUM

ADVAITA IS MATHEMATICAL TRUTH*

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[Editor's Note: Advaita, the principle of Non-duality, is a theory prevalent in the Upanishads. This theory claims that the seen world is unreal, and the only truth of this universe is 'Supreme Consciousness (Paramātma)', which is none else than the individual self (Jivaatma). The particular approach to achieve the state of supreme consciousness is, generally said to be, the process of negation (Apavāda) which leads the aspirant to a state where there is nothing to be seen and this state is generally termed as 'Nish Prapancha Brahma Sthiti' (निष्प्रपंच स्थिति).

To support this approach, they generally repeat the following famous quotation:

अध्यारोपापवादाभ्यां निष्प्रपंचं प्रपंचयेत्

(Adhyaropa Apavadabhyam Nishprapancham Prapanchayet)

One has to pursue the path of (Adhyaropa) super-imposition and (Apavada} negation to achieve and elaborate (Nishprapancha) the Ultra-scenic-state.

In this method, the process of negation is also called 'Pravilapanam' or Annihilation. This term is defined as

'कारण व्यतिरेकेण कार्यस्याभावनिश्चय; प्रविलापनमित्ये तत् '

(The firm conclusion that the effect does not have an existence distinct from that of the cause, is termed as (Pravilapana) Annihilaion..)

This is a generally accepted approach. But, this is not the only approach. Revered Adi Shankara Acharya, who generally advocates this approach in many of his famous works, preaches the other approach in one of his famous minor works called 'Aparokshanubhuti'. He says:

गृह्यमाणे घटे यद्वत् मृत्तिकायाति वै बलात् । वीक्ष्यमाणे प्रपंचेपि ब्रह्मैवाभाति भासुरम् ॥

(When you hold a pot in your hand, the mud comes in to your hand even if you do not want it. In a similar manner, when you see the world, you are essentially seeing the self-luminary Brahman only.)

This approach does not deny the unreality of the seen world, but at the same time, it does not negate the seen objects. In stead, it enforces the aspirant to see Brahman in every object. This is a process of 'Conscious Super-imposition'.

In fact, Bhagavad Geeta, the essence of all Upanishads, upholds both these approaches. In chapter 9, Lord Krishna says:

'मत स्थानि सर्व भूतानि न चाहं तेष्ववस्थित; (श्लो ४)

(All the elements are dependent on Me for their existence, but I am not dependent on them for My existence.)

This statement implies the process of negation.

In chapter 6, He says:

यो मां पश्यति सर्वत्र सर्व च मयि पश्यति । तस्याहं न प्रणश्यामि स च मे न प्रणश्यति ॥

(He, who sees Me in every thing and every thing in Me, will never miss Me and I will never miss him.)

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This statement implies the process of 'Conscious Super-imposition'. This method, renamed as NYASA, is adopted by the TANTRAS,. They proclaim that:

न्यासाभ्यासान्निष्प्रपंचं सत्तत्त्वमनु भूयते ॥

(By the practice of Conscious Super-imposition, NYASA, the "Ultimate Existence" will be directly experienced.)

In contrast to the purely logical approach of the 'Negation Method', the Tantras took up a mathematical approach to prove these propositions, including the ultimate theory of non-duality. H.H. Kalyanananda Bharati Swamy consolidated all such statements and wrote a treatise called "Pūrna Mimāmsā Darśanam" wherein the principles of philosophical mathematics, scattered in the Tantra text books, were arranged in systematic way.

Thus, the Tantras of remote post-vedic period transformed the supreme philosophy into pure science and brought it into the domain of a scientist. Hence Dr. G.S.Murty, a scientist from the cadres of B.A.R.C., Mumbai, enters into this field and applies his knowledge of modern mathematics to magnify, clarify and glorify the ago-old Tantric Mathematics. This being an effort to link and correlate the ancient and modern sciences, we are placing it here, for the perusal of the learned scientists. Editor]

Introduction

Advaita Vedānta is like the mountain Everest in Vedānta. It is not an easy task to reach the peak of Advaita Vedānta. Yet, it is the most fascinating subject, because its message is both intriguing and attractive at the same time. It says there is only One Absolute Brahman. The immediate question that faces us is: 'If there is only One Absolute Brahman, then, what is the multitude of objects seen in the world?' The traditional answer is that the visible multitude of things is 'due to what is called Māyā.' The One Absolute Brahman 'appears' as many entities. If one asks for an explanation of Māyā, the answer is 'it is *anirvacaniya*'; it cannot be explained. It is accepted as an enigmatic power in Advaita Vedānta, which is responsible for the appearance of multitude of things in the world. It seems to be an unsolvable riddle. However, the situation may not be as hopeless as it appears to be.

The riddle of Māyā, in Advaita Vedānta can be re-stated in a slightly different way as follows. One can pose the following question. If there is a 'relation' between Brahman and Māyā, is it like that of ONE and MANY in mathematics? If so, the problem can be shifted to mathematics in stead of answering it in the framework of metaphysics. It is indeed possible to explain Advaita Vedānta by taking advantage of the lead given by Sri Jagadguru Kaḷyāṇānanda Bhārati in his book, entitled *Pūrṇamīmāṁsādarśanam*, in which the Vedic sentence,

'pūrṇamadaḥ pūrṇamidaṁ pūrṇātpūrṇamudacyate ।
pūrṇasya pūrṇamādāya pūṃamevāśiṣyate'

is explained by means of 'geometry of a circle'. This subject is enlarged and discussed extensively in a bilingual book,[1] *Paratattvagaṇ itadarśanam*', which is also called,

[1]Egometry or Principles of Transcendental Philosophy of Mathematical Truth by G.S. Murthy. Publ. Motilal Banarsidass, Delhi. 2002.

Egometry or *Transcendental Philosophy of Mathematical Truth*. It was reviewed in three journals with varying emphasis on its contents.[2]

It is the aim of this paper to offer the essence of *Paratattvagaṇitadarśanam* with minimum possible stress on the arguments in four stages. (1) A few of the important conclusions of *Pūrṇamīmāṁsādarśanam* by Jagadguru Kaḷyāṇānanda Bhārati, regarding Geometry and its relation to metaphysics, will be stated without going into the details. [3] (2) A review of well known mathematical results will also be given without proofs. (3) The intimate affinity between mathematical truths and metaphysical truths will be highlighted. (4) Finally, it will be shown how the meaning of *Mahāvākyās* of *Advaita* can be made visible if one resorts to a geometrical mode of representation of our thoughts. In short, it will be shown there is a path in geometry to go from Paroksha Jnāna to Aparoksha Jnāna.

1.Quotations from Pūrṇamīmāṁsādarśanam

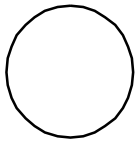
In order to get an insight into the link between metaphysics and mathematics, it is necessary to refer to Pūrṇamīmāṁsādarśanam by Jagadguru Kaḷyāṇānanda Bhārati, in which he explained through a geometrical model the meaning of the Vedic truth:

pūṇamadaḥ pūrṇamidaṁ pūrṇātpūrṇamudacyate । pūrṇasya
pūrṇamādāya pūrṇamevāśiṣyate ॥

"Om. That (Brahman) is infinite and this (universe) is infinite. The infinite proceeds from the infinite. (Then) taking the infinitude of the infinite (universe), it remains as the infinite (Brahman) alone." [4]

The main conclusion of Jagadguru Kaḷyāṇānanda Bhārati, is: "*Purnam, Brahman* or *Satyam-Jnanam-Anantam*, which is One only without a second, has manifested into Perceptible *iswara* whose Visible Form is a Circle" He further writes:

" niravayavaṁ nirguṇaṁ niṣkriyaṁ cinmātraṁ pūrṇam ।
ekamevādvitīyam ।
paridhirāvaraṇam ।



paridhirāvaraṇam ।

vṛ ॥ paridhiḥ maṇḍala rekhā āvaraṇaṁ āvaraṇa śaktiḥ, tathā ca paridhi sadṛśāvaraṇa śaktiḥ ityarthaḥ । yathā bhūmyākāśa saṁdhirūpa paridhi ranantara vastusvarūpa mācchādya tajjñānaṁ pratibadhnāti tadvadāvaraṇa śaktirapi pūrṇajñānaṁ

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[2] See *Ved ānta Kesari*, Vol 92, # 1. pp.43-44, (2005); The Journal of Indian Academy of Mathematics, 26, #2, 2004; and *Vedic Astrology*, 7, #3, 2003.

[3] Swami Kalyanananda Bharati . "*Pūrṇamīmāṁsā Vedānta*". (English)." and (in Sanskrit) "*Pūrṇamīmāṁsādarśanam* ", Virupaksha Sri Peetham, Guntur. Published by Parimi Narayana Sarma Tenali in 1929.

[4] Br. Up. V.1.1.Trans. by Swami Madhavananda. Advaita Ashrama, Calcutta. p. 800.

pratiruṇaddhītyāśaye noktaṁ paridhi rāvaṇamiti । vidyamāna vastvanavabhāsana samarthatva māvaraṇa śakti lakṣaṇa miti bhāvaḥ ॥

vṛttamīśvaraḥ

vṛ ॥ paridhi parivṛta talasvarūpaṁ vṛttaṁ । paridhirnāma āvaraṇa śaktiḥ । talaṁ caitanyaṁ । tathā ca "māyāṁ tu prakṛtiṁ vidyānmāyinaṁ tu maheśvara" mitiśrutyā yathācāvaraṇa śakti rūpa māyāparivṛtatvamīśvarasya tadvatparidhi rūpāvaraṇaśakti yuktatvenavṛttasyeśvara svarūpatva miti bhāvaḥ । "

And, he gave the following Aphorisms also:[5]

etadvai mahopaniṣat ।
tadarthadarśanāt ।
kiṁtarhyasaṁdigdham ।
sāmīpyāt ।
padabījasaṁkhyārekhāṇāmuttarottaraṁ balīyaḥ । "

We shall quote the commentary on the last aphorism:

vṛ ॥ padaṁ śrutismṛtyādi grantharūpaṁ । bījaṁ mantrarūpaṁ । saṁkhyā kādinavet yādi prasiddhā । rekhā cakrādirūpā । tāsāmuttarottaraṁ balīyaḥ । pūrṇajñānaṁ pratipūrva pūrvāpekṣayā uttarottaraṁ balīyaḥ sākṣātkāraṇa mityarthaḥ । nanu saṁkhyāyā ssarvatrān -usyūtatvena tasyā eva kuto na prādhāmyamiti cenna yathā ghaṭādikaṁ prati mṛdassādh- āraṇatvepi kapālānāṁ sākṣāddhetutvaṁ tadvat saṁkhyāyāssarvatrānusyūtatvepy asādhāra ṇakāraṇatvaṁ rekhāyā evetyāvedayituṁ padabījeti sūtramuktaṁ । "

These sūtrās assert the relative importance of padaṁ (Word), bījaṁ (Seed - syllable), saṁkhyā (Number) and rekhā (Line) in the field of metaphysical investigation; and the superiority of one over the other, the highest being rekhā over all others.

With this background, we shall now give the salient mathematical properties of a Circle, needed to link mathematics to metaphysics.

2. Mathematical Properties of a Unit Circle - ROOTS OF UNITY

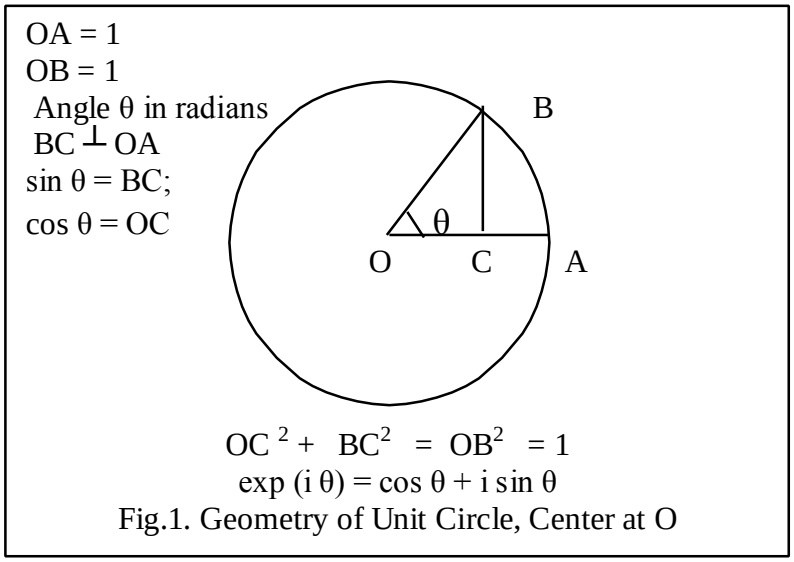
Since we noted that the Supreme is represented by Circle, (*Iswara's* Visible Form is the Circle) one must find out what are those properties of Circle that qualify it to be a visible Form of *Iswara* in addition to the arguments given by the Jagadguru.

The geometrical shape of a Circle is well known, but its mathematical properties are understood only by the students at college level. In particular, we need to understand the concept of a complex number and the meaning of ROOTS OF UNITY, closely related to Unit Circle. The readers who are familiar with that subject can skip this discussion and proceed to the next section.

From a purely geometrical point of view, a Circle is a closed figure drawn on plane paper by means of a compass. It can be represented algebraically as a function, called exponential function of a complex variable, which will be explained

[5] The details can be seen in reference 3, and also in the Appendix 3 of reference 1.

We know the meaning of the angle θ, AOB, shown in the Fig.1 below, in which we assume that OA = OB = 1 are the radii of circle, taken as 1. Such a Circle is called Unit Circle. The point C is the foot of perpendicular BC drawn from B on OA. The exponential function is defined as:



$OC^2 + BC^2 = OB^2 = 1$

$\exp(i\theta) = \cos\theta + i\sin\theta$

Fig.1. Geometry of Unit Circle, Center at O

$\exp z = 1 + z + z^2/2! + z^3/3! + z^4/4! + \ldots.. + z^m/m! +$,
which, becomes when $z = i\theta$,
$\exp(i\theta) = \cos\theta + i\sin\theta$, where, the symbol 'i' stands for $\sqrt{(-1)}$, and cos θ and sin θ are the trigonometric functions and 'i' is the unit of imaginary numbers. Its meaning is: 'i' multiplied 'i' equals '— 1'.
The trigonometric functions cos θ and sin θ are,
$\cos\theta = 1 - \theta^2/2! + \theta^4/4! - \theta^6/6! + \ldots..,$

$= OC;$

$\sin\theta = \theta - \theta^3/3! + \theta^5/5! - \theta^7/7! + \ldots\ldots$

$= BC$

The complex number $\exp(i\theta)$ is represented geometrically in Fig 1. [6]

An important property of the exponential function, $\exp(i\theta)$, also known as Pythagoras theorem, is the result $OB^2 = OC^2 + BC^2 = 1$. This result is stated as '**the modulus of the exponential function exp (i θ) is equal to 1 for all values of θ**' and shown symbolically as

$$|\exp(i\theta)| = 1.$$

This result enables us to understand the meaning of 'Roots of Unity' as illustrated below.

Let us now ask, 'what is the square root of 1'? A mathematician would answer that there are two answers for this question; and they are '1' and '— 1'. Now let us ask, what is the cube root of 1? The answer cannot be given as easily as we can answer the question, what is the cube root of 27 ? We know that 3 is the cube root of 27 becau

[6] The representation of complex numbers as points in the plane is known as Argand Diagram

3 x 3 = 27. However, a mathematician proves that there would be three cube roots of 1 ?'. The argument for this answer is given as follows:

Since the trigonometric functions, cos θ and sin θ, are periodic with a period of 2π, we have the result

exp (2 n π i) = cos (2 n π) + i sin (2 n π) = 1, n=1,2,3

because, sin (2 n π) = 0, for all values of 'n'.

And, if we take cube root of both sides of [exp (2 n π i)] = 1, we get

$[\exp (2 n \pi i)]^{1/3} = 1^{1/3}$, n=1,2,3,

which can be proved to be

$\exp (2 n \pi i /3) = 1^{1/3}$, n =1,2,3,

from the properties of exponential function[7]. Therefore, the three cube roots of unity are:

n=1, exp (2 π i /3) = cos (2 π /3) + i sin (2 π /3)

n=2, exp (4 π i /3) = cos (4 π /3) + i sin (4 π /3)

n=3, exp (6 π i /3) = exp (2 π i) = cos (2 π) + i sin (2 π) = 1.

If these values are potted on the unit circle, they appear as the vertices A, B and C of an Equilateral Triangle inscribed in the Unit Circle as shown in Fig. 2.

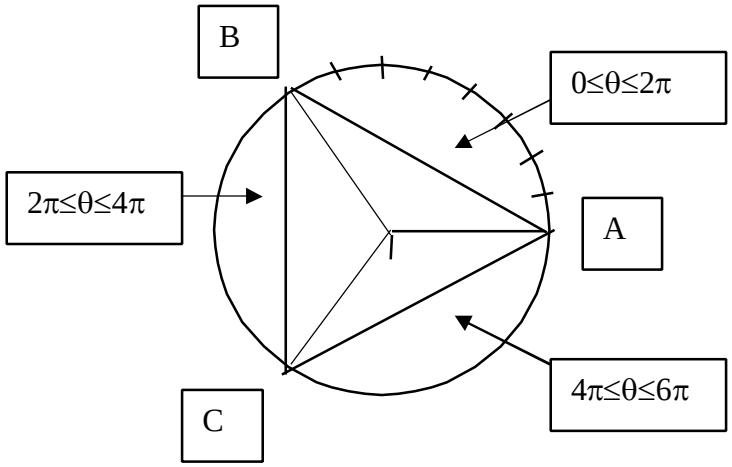


Fig 2.The Circle ABC is Unit Circle. The points A, B and C are cube roots of 1, superposed on Unit Circle. The arc AB of Unit Circle marked by dashes, covers the range 0 - 2π. Similarly the arcs BC and CA cover ranges 2π - 4π and 4π - 6π, respectively. Thus, Unit Circle becomes three arcs by Cube Roots of Unity. Each arc is equal to Unit Circle

In the Fig 2, the arc AB of Unit Circle marked by dashes, represents the function exp (iθ/3) in the range 0< θ < 2π. This means, that the arc AB is a replica of the entire Unit Circle. Similarly the arcs BC and CA cover ranges 2π < θ < 4π and 4π < θ < 6π, each of which is also rep

[7] This is known as De Moivre's Formula/Theorem

the Unit Circle. Thus, the Unit Circle becomes Three Unit Circles by Cube Roots of Unity. Therefore, each of the three PARTS of Unit Circle is equal to the WHOLE Unit Circle.

The proof that "A Part is equal to the Whole" given above is not restricted to the cube roots of 1 only. It is valid for any root of unity. Suppose we want n^{th} root of unity. The unit circle is then 'divided' into 'n' equal arcs and each arc will be equal to the whole circle, what ever 'n' may be. Hence, we have the result that from a mathematical point of view, **'AN ARC OF UNIT CIRCLE IS EQUAL TO THE UNIT CIRCLE'.**
This truth enables us to see the validity of the Vedic statement about Pūrṇam. 'A Unit Circle' is indeed Pūrṇam.

What was said so far is only an 'Illustration' of a Vedic Truth in the mathematical language. Vedic Truths are not amenable to 'empirical proofs'. They are 'transcendental truths' which cannot be grasped by purely logical means. Logical methods may aid us in understanding Vedic truths. Our discussion shows how we can find an example in our mathematical knowledge which echoes a *Vedāntic* Truth. It is necessary for us to examine further if we can understand more *Vedāntic* concepts from our experience in mathematics. The following discussion gives such instances where Mathematics is helpful to illustrate other *Vedāntic* truths.

3. Interpretation of *Avidyā*

There is another fundamental question in *Vedānta* which needs an answer. It can be framed in several ways. We will state it in the following manner. If we accept the *Vedāntic* Truth of the universality of *Pūrnam*, why is it we are not able to comprehend it? The answer in *Vedānta* is that we do not comprehend it because we are subjected to 'a' shielding power called *Avidyā*, which limits our capacity to comprehend *Pūrnam*. If we ask, 'what is the origin of *Avidyā* ?', the answer is that *Avidyā* is primordial, or beginning-less. This is tantamount to saying that we do not have any experience which enables us to comprehend *Avidyā*, just as we could not comprehend Māyā. Under these circumstances, we raise the question, 'Can mathematics shed some light on this fundamental question? The answer is, 'Yes', as shown below.

Let us see the Fig. 3 in which the Unit Circle, AFBCA, is drawn in thin line. An equilateral triangle A-B-C is inscribed in the Unit Circle, and it is also drawn in thin line. Taking AB as diameter, a new circle ADBEA is drawn in thick line. We call the circle ADBE as secondary circle in order distinguish it from Unit Circle. The two circles make up a crescent-shaped figure ADBFA, which is called Lune. The crescent, or Lune, is made up of two arcs, one ADB in thick line and the other, BFA, in thin line. The arc ADB in thick line belongs to secondary circle and the arc BFA in thin line is primary line, since it belongs to Unit Circle. Thus, the crescent or Lune is composed of two arcs, the primary arc and the secondary arc. It has no existence without the triangle A-B-C. The triangle has no existence without the Unit circle in thin line. Therefore, the crescent exists, if and only if, the 'Unit Circle' exists. In the metaphysical language, the Unit Circle is *ādhāra* and the rest is *ādheya*, and we can understand that *ādhāra* is primary and therefore it is *pūrva rūpam* and *ādheya* is secondary, and therefore it is *uttara rūpam* This distinction is also found in *śruti*. Where?

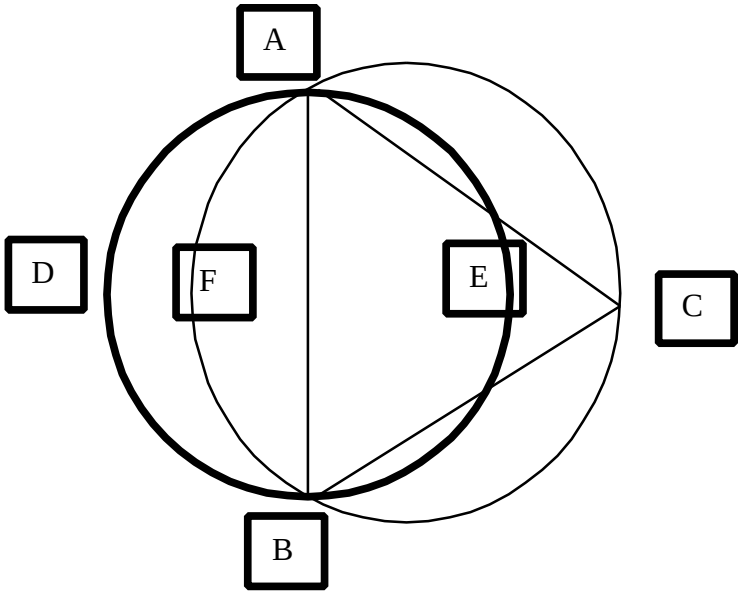


Fig. 3. Illustration of *Avidyā* : AFBCA is Unit Circle. The circle ADBE is drawn with AB as diameter. ADBFA is a crescent formed by two arcs ADB (thick line) and BFA (thin line). Awareness of crescent without the awareness of the Unit Circle AFBC is called Avidyā, a state of incomplete knowledge.

We see in *śikṣāvallī* (Tai. U., I.3,1 - 6) five examples which are designated 'great combinations' (*itīmā mahāsaṁhitāḥ*). We shall restate one of them without going into the sort of detail, which can be obtained, if desired, from published books.[8]

The first one reads:

'1. Athādhilokam । pṛthivī pūrva-rūpam । dyaur-uttara-rūpam ।
ākāśas sandhiḥ । vāyus sandhānam । ity adhilokam ॥

This is translated as follows:

'1. Now with regard to the world (*lokam*): the earth (*pṛthivī*) is the prior form (*pūrvarūpam*), the heaven (*dyau*) is the latter form (uttarūpam), the ether (*ākāśaḥ*) is their junction (*sandhiḥ*), the air is the connection (*sandhānam*). Thus, with regard to the world.

In a similar manner, we can utilize this 'notion' (*bhāva*), and accept the Unit Circle is pūrva-rūpam (*ādhāra*) and the rest is uttara-rūpam (*ādheya*). And in view of the lead given by Jagadguru Kaḷyāṇānanda Bhārati, the notions of *Iśvara* and *Jīva* are defined in '*Paratattvagaṇitadarśanam*' by the Aphorisms:

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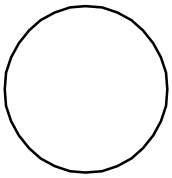
[8] *The Principal Upaniṣads*, S.Radhakrishnan, pp. 528-529.

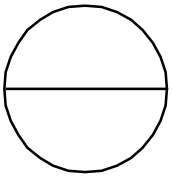
१. अथाधिरेखां व्याख्यास्यामः।
1. athādhirekhāṁ vyākhyāsyāmaḥ।
Now we shall expound the (metaphysical) principles of (geometrical) line.

२. वृत्तं पूर्वरूपम्।
2. vṛttaṁ pūrvarūpam।
Circle (is) the prior form.



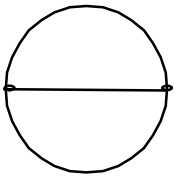
The Circle (is) the prior form.

३. व्यास उत्तररूपम्।
3. vyāsa uttararūpam।
Diameter is the latter form.



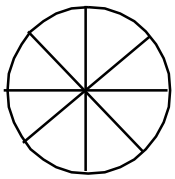
The diameter is the latter form.

४. बिन्दुद्वयं सन्धिः।
4. bindudvayaṁ sandhiḥ।
A pair of points (is their) junction.



A pair of points (is their) junction.

५. अनन्तत्वँ सन्धानम्।
5. anantatvagaṁ sandhānam।
Infinitude (is their) connection.



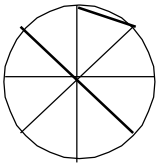
Infinitude (is their) connection

६. एतदीशत्वम्।
6. etadīśatvam।

This is (with regard to) Godhead.

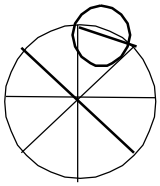
७. अतो जीवत्वम् ।
7. ato jīvatvam ।
Therefore, (the aphorisms on) the notion of Self (are given separately):

८. व्यासः पूर्वरूपम् ।
8. vyāsaḥ pūrvarūpam ।
Diameter (is the) prior form



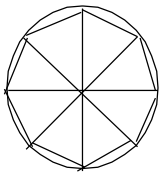
Diameter (is the) prior form.

९. वृत्तमुत्तर रूपम् ।
9. vṛttamuttara rūpam ।
Circle is the latter form.



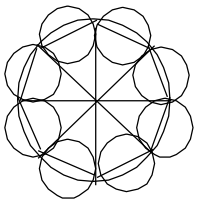
The circle is the latter form.

१०. बिन्दुद्वयं सन्धिः ।
10. bindudvayaṁ sandhiḥ ।
A Pair of points (is) the junction.



A pair of points (is) the junction.

११. अनन्तत्वँ सन्धानम् ।
11. anantatvagaṁ sandhānam ।
Infinitude (is their) connection.



Infinitude (is their) connection.

12. etajjīvatvam ।
12. etajjīvatvam ।
This (is regarding) the notion of the (individual) Soul.
.

१३. इत्यधिरेखा ।
13. ityadhirekhā ।
Thus, (we have expounded) the metaphysical principles
of (the geometrical) line.

These Aphorisms provided the framework for the representation of the metaphysical notions of 'Godhead' and 'individual Soul' in terms of geometrical language in *Paratattvagaṇitadarśanam*.

The Aphorisms 8 and 9 distinguish the notion of jīvatvam by the statements 'Diameter (is the) prior form' and 'Circle is the latter form'. Hence, in the Fig. 3, the circle ADBE represents jīvatvam because it is based on the diameter AB. Hence, the crescent ADBFA is formed by 'intersection' of īśattvam and jīvattvam .

Suppose a jīva has the knowledge of only ADB of the crescent, without the awareness of the arc BFA and the equilateral triangle ABC inscribed in Unit Circle. It is, then, a state of incomplete knowledge of jīva. Knowledge of ADB, without the knowledge of Unit Circle is designated as (a state of) *avidyā*. Therefore, *avidyā* is neither pure ignorance nor pure illusion, but it is incomplete knowledge. It is not *pūrṇa jñānam*. Therefore, avidyā is a concomitant feature, *'ānuṣaṅgika lakṣaṇa*' of māyā, which has the quality of concealing, '*āvaraṇa śakti*'-'vidyamāna vastvanavabhāsana samarthatva māvaraṇa śakti lakṣaṇa miti' as defined by Jagadguru Kaḷyāṇānanda Bhārati. As a corollary, the removal of avidyā of a jīva means the knowledge of the Unit Cirlce which is the pūrva-rūpam (*ādhāra*).

4 (a). Interpretation of *Aham* and *Ahamkāra*

We can represent geometrically the notions of *aham* and *ahamkāra*, as shown in Fig 4, in which the equilateral triangle, A-Ha-M is inscribed in the Unit Circle AFHaGMHA.

It is known from the geometrical relations that the center of symmetry is the same for both the Unit Circle and the inscribed equilateral triangle, which divides the Unit circle into three equal arcs. The Unit Circle represents infinity of points on its circumference. It is devoid of angles. This feature corresponds to *nirguṇatvam*. The inscribed triangle is represented by three points of intersection of three lines and hence it represents finiteness, the triad of points of the infinite. An entity which is finite is known as *vyakta*, and it is individualized. A conscious individual refers to itself (himself or herself) by the word 'aham'. Hence the equilateral triangle inscribed in Unit Circle is represented by the word 'A-Ha-m'. And it is an individualized form of the Infinite. Since the Unit Circle is Primary, we designate it by a word AHam the Primary Individualization.

We noted before that the equilateral triangle AHaM gave rise to the secondary circle ADHaEA, shown in thick line, with AHa as its diameter. Since the cord AHa is less than the Diameter of the Unit Circle, we called the circle ADHaEA in thick line as a secondary circle (uttara rūpam). The secondary circle extends, of course partially, beyond the circumference of the Unit Circle. The extension of secondary circle beyond the Unit Circle is shown by the hashed area in Fig.4. It forms the crescent we disc
before in conjunction with the Unit Circle. This crescent is not independent of th
Cirlce, but because of *avidyā*, **incomplete knowledge** (See Fig. 3), the crescent fe

be an independent entity. This feeling of independence, arising out of incomplete knowledge, manifests as *ahamkāra* of an individual conscious being.

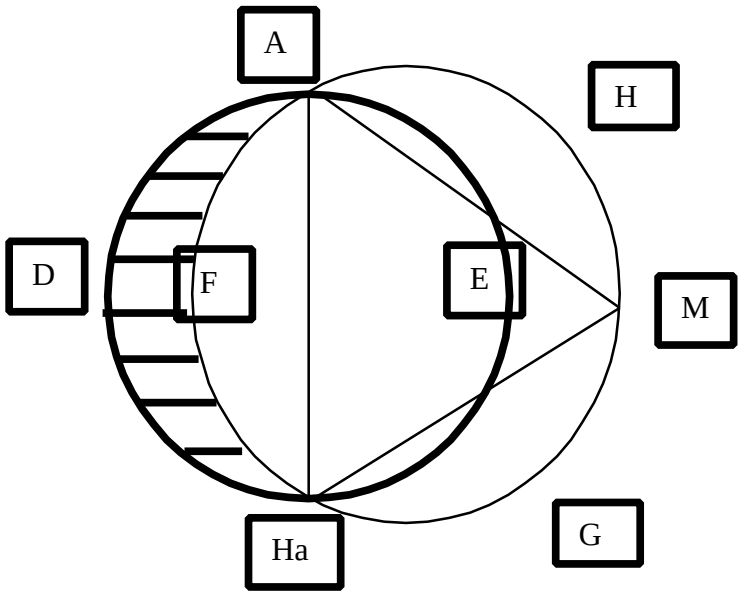


Fig. 4. Illustration of aham and ahankāra. AFHaGMHA is Unit Circle. The equilateral triangle A-Ha-M represents individualized consciousness. The crescent (hashed area) ADHaFA formed by two arcs ADHa and HaFA represents ahankāra arising out of avidyā, which is awareness of crescent without the awareness of the Unit Circle. avidyā and ahankāra are inseparable. They are like arcs of a crescent, representing the dichotomy of a jīva-īśvara.

Jagadguru Adi śaṇkarācārya defined ahamkāra as: 'avyakta associated with avidyā is ahamkāra'.[9] This definition is understood mathematically as follows: The word *avyakta* indicates a state of non-individualization, which is represented by the Unit Circle. Further, we understood that the word avidyā stands for incomplete knowledge, which is represented by the crescent. The inscribed equilateral triangle AHaM is the link between the two, the crescent and the Unit Circle. So, the sequence in this chain is: (1) the Unit Circle, (2) AHaM, the equilateral triangle inscribed in the Unit Circle, and (3) the diameter of the secondary circle, AHa, and (4) the crescent formed with the Unit Circle and the secondary circle.

With these definitions, we are able to see the meaning of *ahankāra* in terms of the geometrical illustration of avyakta and avidyā : "*avidyā samyukta avyaktam ahamkārarah*.

[9] See the commentary on The Bhagavadgita Ch. VII Sl.4 by Jagadguru Adi śaṇkarācārya.

4 (b) Three roots of unity and Three individualized States of Consciousness

We saw in Fig. 4, that the Unit Circle has an equilateral triangle inscribed in it denoted as AHaM. One side of the equilateral triangle, A-Ha, gave rise to the secondary circle. Similarly, the remaining two sides of the equilateral triangle, Ha-M and A-M can give rise to secondary circles, as shown in thick lines in Fig. 5. Each of the secondary circle can have an inscribed equilateral triangle (not shown in Fig. 5). We denote these secondary equilateral triangles by the name, 'a-ha-m'. Such notation has the following advantage.

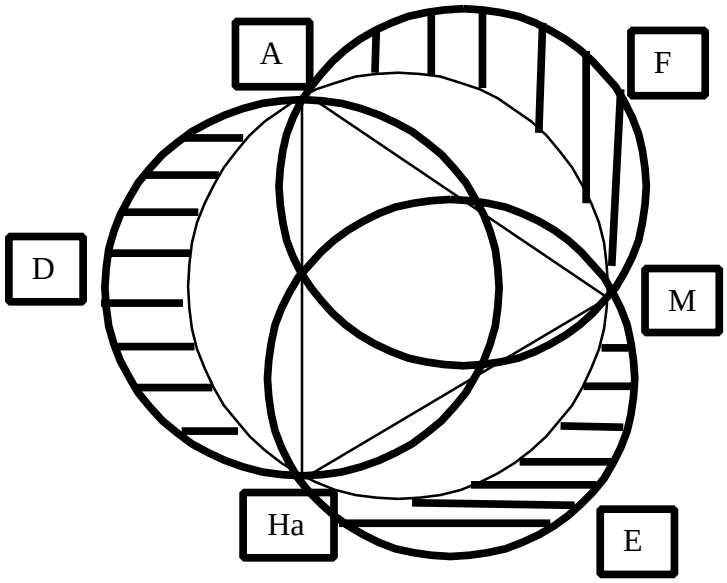


Fig. 5. Illustration of three ahamkāras . The three Cube roots of unity are shown as three vertices of the equilateral triangle A-Ha-M inscribed in Unit Circle in thin line. Taking each of the three cords, A-Ha, Ha-M and M-A as diameter, three secondary circles are drawn in thick lines, representing three jivas. Each secondary circle is endowed with Individualized consciousness. The three crescents (hashed areas) A-D-Ha, Ha-E-M and M-F-A represent three ahamkāras arising out of avidyā of the three jivas.

We can easily see that the secondary circle is smaller than the Unit Circle because the cord A-Ha in the Unit Circle is less than the diameter of the Unit Circle. Thus, there is a difference between secondary circle and the Unit Circle. We saw that this difference is utilized in the *Paratattvaganitadarsanam* to define jīva and īśvara; the secondary circle represents a jīva, since the Unit Circle represents Isvara.[10]

Therefore, we arrive at the result that there exists a difference in AHam and aham. AHam can exist in the Unit Circle and there is no AHamkāra in the Unit Circle. But there would be both 'aham' and 'ahamkāra' in the secondary circle. The metaphysical implication is that there can be no avidyā for *īsvara*, who is omniscient (sarvajñah)

We arrive thus at an important metaphysical conclusion that a jīva has both 'aham' (ego) and ahamkāra (egoism). .

Further, it can be seen that jivatvam does not arise when Unit Circle is divided into two parts due to square root of unity. The reason is that in the case of square root of one, the Unit Circle becomes two Unit Circles which are superposed on each other and are indistinguishable from each other; and hence there is no secondary circle and so, no ahamkāra can arise.

It follows, therefore, that 'aham' (ego), ahamkāra (egoism) and avidyā arise only if Unit Circle is divided in three or more parts, that is, if we look for three, or more than three, roots of unity.

4(c). Tattvamasi

Now, we shall explain the mathematical representation of the *Mahā vākya*, *Tattvamasi* based on the previous discussions. The steps in the argument are as follows.

We showed already that the cube roots of unity divide Unit Circle into three equal parts each of which is the Unit Circle. In the Fig 6, the arc A-F-Ha is identical with the Unit Circle, The crescent A-D-Ha-F-A represents avidyā. The cord A-Ha and the arc A-F-Ha are inseparable. The arc A-F-Ha is denoted as *TAT*, meaning THAT (Brahman), the implied meaning of which is the 'pūrṇam'.

Further, the Teacher who realized the metaphysical truth of *Tattvamasi* points to the arc A-D-Ha identifying it with the individual consciousness associated with avidyā, and addresses the student by the word *TVAM* meaning YOU. Since the arc A-F-Ha is inside the arc A-D-Ha, the Teacher says to the student '*Tvayi Tadasti*', meaning 'THAT is in YOU'. After showing that A-Ha, the diameter supports the secondary circle, and that it is inside the Unit Circle, the teacher 'shows' to the qualified student the meaning of *Tasmin Tvamasi* (You are in That). When the two sentences *Tvayi Tadasti* and *Tasmin Tvamasi* are explained, the Teacher finally utters the sentence, *TASMAT TATTVAMASI*, which 'triggers' the internal instruments, *antah karaṇa*, of the student into activity and leads him/her to realize the import of the *Mahā Vākya* – 'aham Brahmāsmi' in the form bāḍhaṁ, tadasmyaham , 'Oh, Yes, I am That'. This is a step in between Tattvamasi and Aham Brahmāsmi. The former is paroksa jñāna and the latter is aparoksha jjñāna. Thus,

[10] See *Jiveswaradhikaranam* in *Paratattvaganitadarsanam*; Reference #1

the mathematical representation of vedānta provides an intermediate step between paroksa jñāna and aparoksa jñāna.

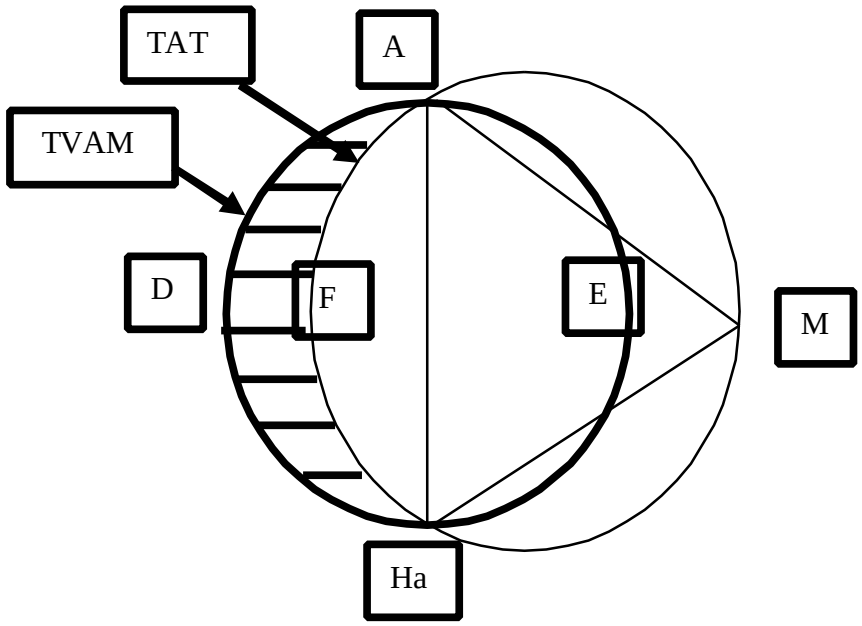


Fig.6. Illustration of *TAT-TVAM-ASI*. An equilateral triangle AHaM is inscribed in Unit Circle AFHaMA. The crescent (hashed area) ADHaFA formed by two arcs ADHa and HaFA represents *Ahamkara* arising out of *Avidya*. Owing to the property of Roots of Unity, the arc AFHa is 'equal' to the entire Unit Circle. Further, the arc AFHa is 'inside' the secondary circle marked in think line. Further the cord AHa is inside the unit circle, and it is also the diameter of the secondary circle. The arc AFHa represents TAT (Unit Circle) which is pūrnam and arc ADHa represents TVAM (Individualized Consciousness). Based on these truths, the Teacher who is *Brahmavit* 'shows to the qualified student: "Therefore, YOU are THAT – *Tasmat Tattvamasi*"

Discussion

The preceding explanations reveal that metaphysical truths can be articulated in a mathematical language and can supplement other methods. Since the modern civilization is built on the knowledge of mathematics, one can hope that a large section of modern scientists can benefit from this method. The traditional methods followed by our predecessors are no doubt well founded on firm logic, but they are not amenable for easy comprehension. Only students as meritorious as Swami Vivekananda can gras message of the Master Sri Ramakrishna: ''Kali is none other than Brahman. That

is called Brahman is really Kali".[11] For most people, it needs very specialized training in subtle logic. The method of *adhyāropa-apavāda* is too elaborate to be understood by a modern scientist, because it needs primarily an understanding the principle of superimposition of unreality on the Real.[12] It is called *Adhyāsa*.[13] This is followed by *apavāda*, which is the realization of the Real like the realization of the rope falsely understood as snake. Though this is a useful method as far as it goes, the modern mind seeks to know, 'How does one decide the falsity unless the real is known?' There is thus a Gordian knot[14] facing the modern scientist. This knot is cut with the help of mathematics which provides a firm ground for the validity of statement that 'a part is equal to the whole'. Once this is understood, the route to the statement *Tattvamasi* is free from further hurdles. The knowledge of mathematics shows the way. Geometry is both a telescope which brings into view the distant object, and a microscope which magnifies the small object, and brings into view both objects into the region of direct experience, that is, *aparokshajñāna* . There is nothing to falsify but there is a need for correct understanding. It is a question of placing things in proper perspective. In a sense, it is *sannyāsa*, understood as s*amyak nyāsa* – proper placement (of experience)**.**

Advaita Vedānta has a long history.[15] The two words *avidyā* and *adhyāsa* still occupy the central place in the post-Sankara period to convey the message of Vedānta.[16]

Before we finally end this discussion, it is proper to stress that the relation between Non-Dual Brahman and Māyā, is like the relation between the Unit Circle and the Roots of Unity, reminding us the message of Lord Krishna in the Srimad Bhagavadgita, VII-7:

mattaḥ parataraṁ nānyatkiṁcadasti dhanaṁjaya ।
mayi sarva midaṁ protaṁ sūtre maṇi gaṇā iva ॥

"Beyond Me, O Dhananjaya, there is naught. All this is strung in Me as a row of jewels on a thread".[17] The analogy of 'row of jewels strung on thread' is noteworthy. The Roots of Unity on the Unit Circle are like row of jewels on a thread.

The intimacy of the Transcendental Being and the Empirical Individualized conscious being is revealed in several scriptures. In Srimad Bhagavatdgita, we see the verse:

samohaṁ sarva bhūteṣu na me dveṣyao'sti na priyaḥ ।
ye bhajanti tu māṁ bhaktyā mayi te teṣu cāpyahaṁ ॥

"I am the same to all beings: to Me there is none hateful or dear. But those who worship me with devotion, are in Me and I too am in them".

In this verse, the phrase, 'But those who worship me with devotion, are in Me and I too am in them' reiterates the basic truth of Advaita.

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[11] See, *The Gospel of Sri Ramakrishna*. Translation by Swami Nikhilananda. N.Y.!942, p.734.
[12] See *Vedāntasāra* of Sadānanda. Translation by Swami Nikhilananda, Advaita Ashrama 1931.Page 83
[13] See *jijñāsādhikaranam* of *Brahmasūutra Bhāshyam* by Jagadguru Adi Shankaracharya. One can see on the internet a large number of articles on Adhyāsa.
[14] Any perplexing problem.
[15] *A Survey of the Pre-Sankara Advaita Vedanta*. M.T. Sahasrabudhe. University of Pune, (1968)
[16] See for example ; *Avidya and Adhyasa, Veiling and Projecting*. Swamy Jnaneshvara Bharati. (Homepage, Internet).
[17] Translation by Swami Swarupananda. Advaita Ashrama. Calcutta.

The reason why we do not feel the truth of Advaita is not that it is difficult to grasp, but we do not pay due attention to the words of those who understood the truth of Advaita. In fact, the fundamental requirement for Aparoksha Jnāna is stated in śruti.[18]

naiva vācā na manasā prāptuṁ śakyo na cakṣusā ।
astīti bruvato'nyatra kathaṁ tad upalabhyate ॥ (Kaṭha Up. II.3.12)

"It cannot be attained through speech, nor through mind, nor through eye. How can it be known to anyone apart from him who speaks of It as existing?

astītyevopalabdhavyastattva bhāvena cobhayoḥ ।
astītyevopalabdhasya tattva bhāvaḥ prasīdati ॥ (Kaṭha Up. II.3.13)

"The Self is (first) to be realized as existing, and (then) as it really is. Of these two (aspects), the real nature of the Self that has been known as merely existing , becomes favourably disposed (for self-revelation)."

The nature of geometry is not yet fully explored.[19] Its importance for metaphysics was stressed by Jagadguru Kaḷyāṇānanda Bhārati. In his book referred to earlier, he encapsulated metaphysics in geometry and calibrated it by his Aphorism: Circle is Godhead.

The importance of geometry was recognized by Jagadguru Adi Śaṇkarācārya also. In all the centers where His disciples are following His teachings, the worship of Sri Yantra is still practiced. It is sufficient to indicate that Sri Yantra is held in great esteem in Advaita Philosophy. It is a curious fact that an exposition of the philosophy of Sri Yantra is not as popular as the exposition of Advaita Vedānta. As a matter of fact, the Sri Yantra, seen as a purely geometrical figure, has some gross features which are very attractive. It contains only nine triangles intersecting each other to form a very beautiful shape with profound symmetry. It has left-right symmetry but not top-bottom symmetry. In this respect it is like a crescent which has only left right symmetry but no top-bottom symmetry. If we imagine a line of symmetry is drawn in crescent in Fig.3, it passes through the center of the Unit Circle. It is tempting to examine whether the model of a Jiva which is constructed in this discussion may be extended to the metaphysics of Sri Yantra, which is considered by followers of Sri Vidya as a 'genetic code of the cosmos and the individuation'.[20]

We have given a few examples of how geometry enables us to grasp metaphysical truths. There are, however, other features discussed in *Paratattvaganitadarsanam* and interested readers can refer to that book.

I am indebted to Professor Pullela Sri Ramachandrudu garu, and Professor P.G. Lalye , (Retd) Professors of Sanskrit, Osmania University, Hyderabad, and Brahma Sri Kuppa Venkata Krishna Murthy garu, Chairman and Managing Trustee , I-SERVE, Hyderabad for very useful discussions.

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[18] *Eight Upanishads*. Vol I. Swami Gambhirananda.Advaita Ashrama.

[19] *Theory Behind Mathematics*. G.S.Murty. National Conference on 'Vedic knowledge: Contempora Relevance'. June 16-18, 2005. Organised by JNIAS and I-SERVE, Hyderabad

[20] G.S.Murty. *Śrī Cakram - Its Geometry and Metaphysics*. Publ. Pranav Bharati Foundation, Ahm . Comments of Amrutananda Natha on:p. 12..

Harih OM Tat Sat

Sarvajit Aasvayuja Suddha Navami/Dashami

Octobrt 17/18, 2007, Dashara

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